Small-scale advection and the neutral wind profile

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The flow of air in the lowest few metres of the atmosphere is examined in the case when a neutrally stratified logarithmic profile encounters a sudden change in surface roughness. Examination of the available evidence suggests that the ratio of new to old friction velocity is given by the ratio of the roughness lengths raised to a power about equal to 0·09. Some field observations and wind-tunnel measurements over a change in roughness indicate that vertical displacement of the streamlines at all heights within the range considered commences very near the surface transition. Calculations are made of the fetch necessary for the new wind profile to be established up to a given height and indicate that its ratio to the height is not constant but varies with the height and amount of change in roughness. Over a fairly wide range this ratio is about 100–150.

1. Introduction

A good deal is now known (see, for example, Priestley 1959) about the dependence of the vertical turbulent fluxes of heat, water vapour and momentum in the lower atmosphere on the corresponding vertical gradients of temperature, humidity and wind speed in a wide variety of stability conditions. Most theoretical treatments, however, assume that the problem is one-dimensional, i.e. that the flow is horizontally uniform.

In practice, however, fluxes must be deduced from gradient measurements made over sites of limited horizontal extent. Little appears to be known of the height up to which gradient observations can safely be made for any given fetch of uniform upwind terrain and applications of the theories have at times fallen into error for this reason.

This paper attempts to investigate the effects of advection in the case where most is known of flux-gradient relationships—that of the wind profile in neutral conditions of stability. This is a case where the approach of Philip (1959), who considers advection as a problem in diffusion, cannot be used. He takes the advection to be a consequence of changes in boundary conditions but does not permit these changes to effect any variation in the magnitude of the turbulent transfer coefficient which is thus constant throughout the whole flow: in neutral wind profiles this coefficient is determinate in terms of (*inter alia*) the surface shearing stress and therefore must change with the roughness of the boundary. The present approach is dynamical, rather than by way of a diffusion equation.

A previous attempt to treat the same question dynamically has been made by Glaser, Elliott & Druce (1957). (This is in the nature of a final report and embodies earlier papers by Glaser 1955*a*, *b* and by Elliott 1958.) In their work, a wind flow in neutral equilibrium, displaying the usual logarithmic velocity profile, is allowed to encounter a sudden change in surface roughness. From this point, an 'internal boundary layer' is supposed to develop having a clearly defined top across which the wind velocity is continuous and the stress discontinuous. Above and upwind of the internal boundary layer, the parameters of the original flow apply. Inside it, the developing logarithmic profile is defined by the new roughness length and a new shearing stress which, while constant with height, is allowed to vary with downwind distance in such a way as to maintain the required continuity of velocity at the top of the layer.

This theory appears on consideration to have two main shortcomings. The downwind variation of shearing stress which it requires after the surface transition is, in fact, quite large. (See figure 2–2 of the paper by Glaser *et al.* 1957.) This is at variance with wind-tunnel observations by Jacobs (1939) which suggest that a new constant value of shearing stress at the surface is taken up almost immediately after a change in roughness. Secondly, their figure 1–2 indicates that the new wind profile is fully established after a downwind travel of some 10–20 times the height of observation. Most micro-meteorologists would expect, out of the general background of their experience, that a rather greater distance than this is required to establish a new profile.

2. Theoretical considerations

We consider a situation as sketched in figure 1. The wind is taken as blowing from left to right with a fully established profile in region 1. At A a change occurs in surface roughness and in region 2 a new profile, with constant shearing stress,

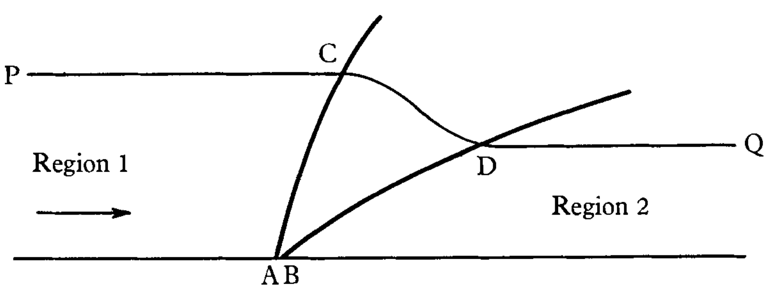

FIGURE 1. Scheme of transition considered.

is again established. AC and BD, marking the boundaries of the transition region, represent the limits between which the stress is varying and are thus lines of constant stress. If the surface to the right of A is smoother than that to the left, PCDQ may resemble a typical streamline. No assumptions are yet made as to the size or shape of the region ABDC.

Variation of wind direction with height and with roughness will be neglected and the problem treated as two-dimensional. The density, ρ, may also be taken as constant and we have

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=\frac{\partial}{\partial z}u_*^2-\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{1}$$

as the equation of horizontal motion. Here x and z are the horizontal and vertical co-ordinates respectively, u and w are the corresponding velocity components, u_* is the friction velocity (defined as the square root of shearing stress divided by density) and p is the pressure. From the equation of continuity, a stream function ϕ_1 can be defined

$$\frac{\partial \phi_1}{\partial z} = u, \quad \frac{\partial \phi_1}{\partial x} = -w,$$

and we have

$$\frac{\partial \phi_1}{\partial z}\frac{\partial u}{\partial x} - \frac{\partial \phi_1}{\partial x}\frac{\partial u}{\partial z} = \frac{\partial}{\partial z} u_*^2 - \frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{2}$$

Integration of equation (2) over the region ABDC gives

$$\int_0^\phi (u'' - u')\, d\phi_1 = -\oint u_*^2\, dx - \rho^{-1} \oint p\, dz$$
$$= -X - Y, \tag{3}$$

where single and double primes are used to distinguish quantities in regions 1 and 2 respectively, ϕ is the value of the stream function along CD, the velocity difference $(u'' - u')$ is to be considered as taken along a streamline and the circular integrals are taken in the direction ABDC.

We consider now the relative magnitudes of the terms X and Y of equation (3). Provided that the horizontal distance between A and B is much less than that between C and D, X can be put in the form

$$X = \eta(u_*'^2 - u_*''^2),$$

where η is the distance in the x-direction between A and some point between C and D which, provided that u_* varies fairly regularly along CD, is probably about midway.

To estimate Y, note that, between C and D, w is of order $u(z'' - z')/(x'' - x')$, where (x', z') and (x'', z'') are the co-ordinates of C and D. Since, in moving from C to D, the air must be accelerated vertically to a maximum of $|w|$ and then decelerated again, dw/dt is of order $u^2(z'' - z')/(x'' - x')^2$. This is very much less than g as is also any reasonable estimate of $\partial u_*^2/\partial x$. The pressure can thus be taken as hydrostatic everywhere. Since the gradient of surface level pressure can be considered constant over distances of a kilometre or so, $\partial p/\partial x$ can be considered constant everywhere. We therefore have that

$$Y = \frac{1}{\rho}\frac{\partial p}{\partial x} \times \text{(the area ABDC)}$$
$$\approx \frac{1}{\rho}\frac{\partial p}{\partial x}\frac{z(x'' - x')}{2},$$

where the area ABDC has been considered approximately triangular. It will be pointed out later that C is almost vertically above A so that $(x'' - x') \approx 2\eta$ and it follows that

$$\frac{Y}{X} = \frac{z\, \partial p/\partial x}{\tau'' - \tau'},$$

where τ, the shearing stress, equals ρu_*. For realistic values of $\partial p/\partial x$, this ratio is much less than 1 provided that z does not exceed a few metres and $\tau''-\tau'$ does exceed a few tenths of 1 dyn cm^{-2}. From equation (3) we can thus write, in most cases,

$$\int_0^\phi (u''-u')\,d\phi = \eta(u_*'^2 - u_*''^2). \tag{4}$$

It is to be noted that the equation of motion gives no information about the size of the transition region but only about the location of some roughly central line in it. No details of wind profile have been assumed so far so that equation (4) is valid for all stability conditions.

Restricting ourselves now to neutral conditions, the wind profile is given in the usual form by

$$u = (u_*/k)\ln(z/z_0), \tag{5}$$

where k is von Kármán's constant, equal to about 0·4, and z_0 is the roughness length. From equation (5) it is easy to show that, if $\phi = 0$ at $z = z_0$,

$$\phi = (u_* z/k)\{\ln(z/z_0) - 1\} \quad \text{for} \quad z/z_0 \gg 1, \tag{6}$$

and

$$\int_0^\phi u\,d\phi_1 = zu^2 - 2\phi u_*/k, \tag{7}$$

so that, given u_*', u_*'', z_0' and z_0'', η can be evaluated immediately from equation (4) without any integrations.

3. The new shearing stress

Not all of the four quantities u_*', u_*'', z_0', z_0'' upon which η depends can be considered independent. In any practical case, the two roughness lengths and one of the friction velocities (or what is equivalent, the actual velocity at a given height) are likely to be specified and the other friction velocity must be regarded as thus determinate. In what follows the value of u_*' will be supposed given and u_*'' sought.

The appropriate condition to apply is that the flows in the two regions take place under the same geostrophic wind. In considering momentum exchanges in the lowest few metres it is customary, and justifiable, to neglect the Coriolis force, f, and this has been done above in equation (1). However, when we are concerned with the relationship between the flow in the shallow surface layer and that at much greater heights, this neglect is no longer allowable and f must be admitted as a relevant variable. In a barotropic friction layer in neutral equilibrium, the friction velocity can depend only on f, z_0 and the magnitude u_g of the geostrophic wind. In non-dimensional form, then, we must have

$$u_*/fz_0 = F(u_g/fz_0).$$

There exist two detailed theoretical studies of the velocity and stress distributions in such a friction layer. The earlier is by Rossby & Montgomery (1935) who consider a two-layer model. The lower of these is the usual constant-stress layer with logarithmic profile, mixing length given by $l = k(z+z_0)$ and the turbulent transfer coefficient given by $K = ku_*(z+z_0)$. In the upper layer they take

$l = k_1(h-z)/\sqrt{2}$, where k_1 is a constant estimated as equal to 0·065 and h is the height at which the geostrophic wind is attained. In this layer they deduce that the transfer coefficient is given by $K = f(h-z)^2/3\sqrt{2}$. They then assume continuity in u_*, l, wind velocity and wind direction at the junction of the two layers and give expressions from which it is possible to calculate u_*/fz_0 as a function of u_g/fz_0.

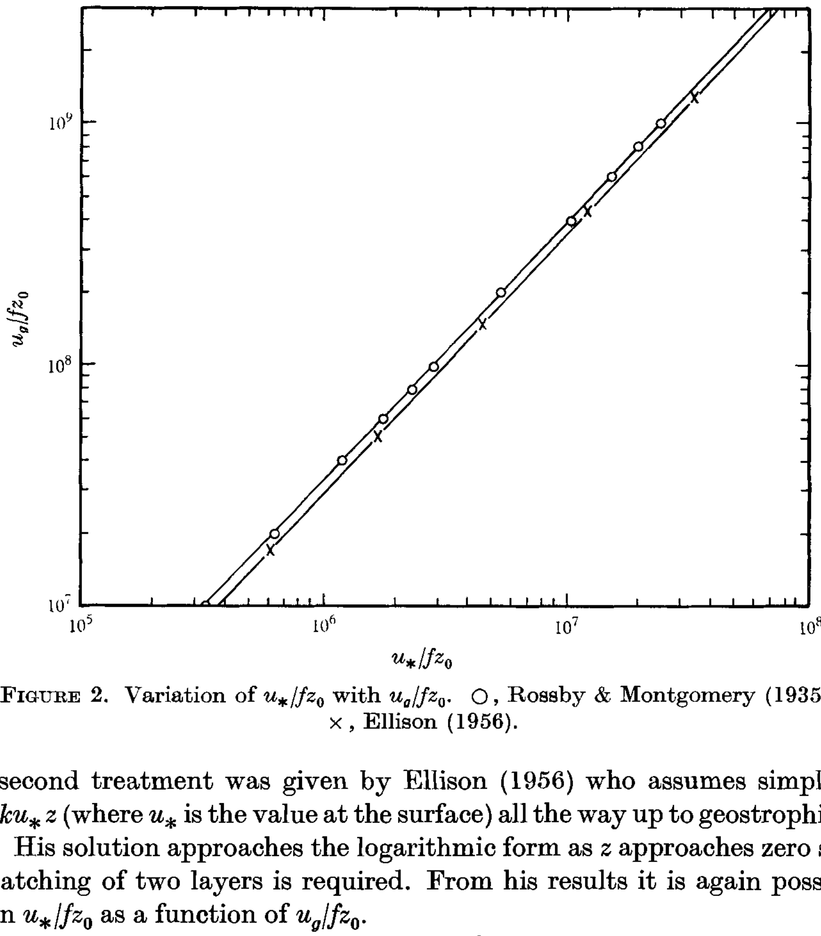

FIGURE 2. Variation of u_*/fz_0 with u_g/fz_0. ○, Rossby & Montgomery (1935); ×, Ellison (1956).

A second treatment was given by Ellison (1956) who assumes simply that $K = ku_* z$ (where u_* is the value at the surface) all the way up to geostrophic wind level. His solution approaches the logarithmic form as z approaches zero so that no matching of two layers is required. From his results it is again possible to obtain u_*/fz_0 as a function of u_g/fz_0.

Figure 2 shows the results as calculated from these theories. The two agree closely in showing u_g/fz_0 proportional to $(u_*/fz_0)^{1·071}$ and hence, with u_g and f constant, u_* proportional to $z_0^{0·066}$. Even the constants of proportionality given by the two theories differ by only about 15 %. This agreement is most surprising when one considers the very great difference between the two K-distributions assumed and suggests that the solutions may be determined more by the boundary conditions—geostrophic wind above, logarithmic profile below—which are common to both than by the details of the mixing in between. It is tempting to extend this argument to conditions of other than neutral stability because even there the profile tends to the logarithmic form as the ground is approached. It is clear, however, that this cannot be valid because constancy of geostrophic wind over, say, 24 hr would then imply constancy of u_* over the same period and, in consequence, the variation of wind velocity near the surface (but high enough to be affected by the stability) would be in the wrong sense between day and night. The above proportionality between u_* and $z_0^{0.066}$ can therefore only be expected to apply in the neutral conditions for which it was derived.

Since there is good theoretical backing for an expression of the form

$$(u_g/fz_0) \propto (u_*/fz_0)^m,$$

any observational evidence on the variation of u_* with u_g at constant f and z_0 can be used to deduce that of u_* with z_0 at constant f and u_g. Some such observations are given by Lettau & Davidson (1957). In all, there are 23 pairs of nearly simultaneous measurements of u_* and u_g made when the absolute value of the Richardson number at 1·6 m was equal to or less than 0·01. These showed u_g to be proportional to $u_*^{1.062}$. Unfortunately, the standard error of this index is quite high, being 0·27. The variation of u_g as $u_*^{1.062}$ implies that u_* is proportional to $z_0^{0.058}$.

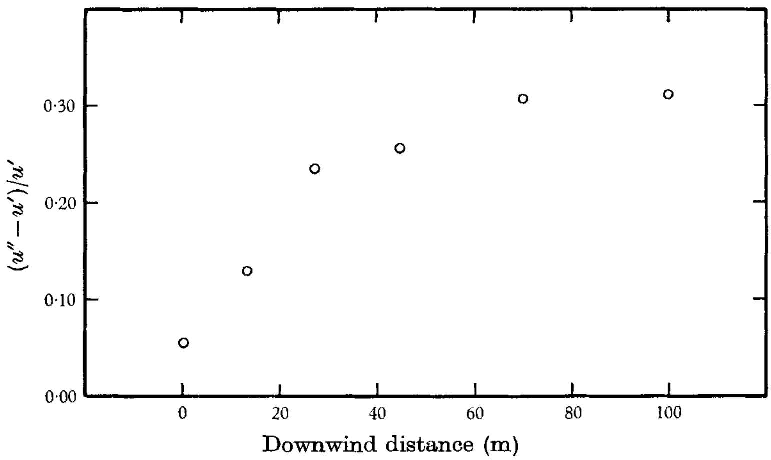

FIGURE 3. Variation of wind speed in moving from long to short grass.

Further information can be obtained from some unpublished observations made by E. L. Deacon in 1945 in near-neutral conditions and a wind speed of 4 to 5 m sec^{-1}. Unfortunately the details of the observations are now lost and the summary presented in figure 3 is the only part, relevant to the present issue, which survives. Measurements of wind speed, u'', were made at a height of 1 m over short grass (1–2 cm high) at various distances downwind of the edge of a large area of long grass (about 50 cm high). They were compared with a simultaneous measurement of the velocity, u', at 1 m over the long grass itself. Each point in figure 3 shows the average of four five-minute observations of $(u''-u')/u'$. It would appear that the limiting value of $(u''-u')/u'$ at very large downwind distances might be taken as about 0·35. If we put $u_* \propto z_0^n$, then, from equation (5), the limiting value of this ratio is given by

$$\frac{u''-u'}{u'} = \left(\frac{z_0''}{z_0'}\right)^n \frac{\ln(z''/z_0'')}{\ln(z'/z_0')} - 1.$$

Heights of observation were 100 cm for both u' and u'' but, while it is proper to put $z'' = 100$ cm, it seems more reasonable to allow a zero-plane displacement in the case of z'. Fortunately the calculation is not very sensitive to the exact value chosen and 10 cm has been used in what follows. No direct determination of z_0' and z_0'' is possible but some estimate can be made (Deacon 1953). The minimum and maximum reasonable values of z_0' may be taken as 4 and 6 cm and those of z_0'' as 0·2 and 0·4 cm. By combining opposite extremes, values of n and some idea of its sensitivity to the choice of z_0' and z_0'' can be obtained. Taking $z_0' = 6$ cm, $z_0'' = 0{\cdot}2$ cm, gives $n = 0{\cdot}156$ and $z_0' = 4$ cm, $z_0'' = 0{\cdot}4$ cm gives $n = 0{\cdot}120$. Obviously there is no very great sensitivity to the exact choice of z_0 and the mean of the two results, 0·138, may be taken as a reasonable estimate of the probable value of n from Deacon's observations.

4. Wind-tunnel observations

The evidence of Jacobs (1939) that the new surface shearing stress is attained very soon after a change in roughness is not wholly convincing in that the stress determinations depend on an assumed mixing-length distribution over the tunnel section, although the distribution assumed does not appear unreasonable at small heights. To obtain further information on this point and also on the extent of the transition region of figure 1, a new series of wind-tunnel measurements was undertaken.

Use was made of an existing small wind tunnel by building for it an extended working section 250 cm long. The height was 17·0 cm and the breadth 14·0 cm. A vane air meter was mounted in a fixed position at the downwind end of the working section in order to provide a reference velocity and this, together with the size of the carriage carrying the pitot and static tubes, reduced the length available for velocity exploration to 230 cm.

In the atmosphere, it is reasonable to consider a fully-developed profile meeting a change in surface roughness. In the wind tunnel, however, the boundary layer will continue to develop downstream, a process limited only by the ultimate establishment of pipe flow if the tunnel is long enough. Hence it is necessary to determine the velocity field twice; once with the whole length of the tunnel floor having the roughness z_0' appropriate to region 1 and again with a transition from z_0' to z_0'' imposed. In this way it is possible to determine the effect of the transition in displacing the streamlines from the positions they occupy in its absence.

The two determinations of velocity field, which will be referred to as case 1 and case 2 respectively, were made: (*a*) with the whole tunnel floor covered by 16-grit garnet paper, and (*b*) with about 100 cm of this paper followed by 150 cm of 40-grit garnet paper. Both of these abrasive papers are standard commercial products. Velocity profiles were determined at a free-stream velocity of about 5 m sec^{-1} and, in general, at 10 cm intervals of downwind distance. There were two exceptions to this spacing; for the last 80 cm of case 1 measurements, where the streamlines were nearly horizontal, 20 cm intervals were found adequate and, in case 2, two intervals of 5 cm were inserted near the transition point to give better horizontal resolution.

Velocity profiles in the form of equation (5) were fitted to the observed wind speeds in the lowest few millimetres and values of u_* and z_0 derived. The results indicated that in case 1 the flow was aerodynamically rough and that stresses determined in this way are therefore acceptable. The flow in case 2 was, however, aerodynamically smooth and the stresses were therefore redetermined from the appropriate wind profile equation

$$u/u_* = k^{-1} \ln \{(u_* z/\nu) + 5{\cdot}5\}.$$

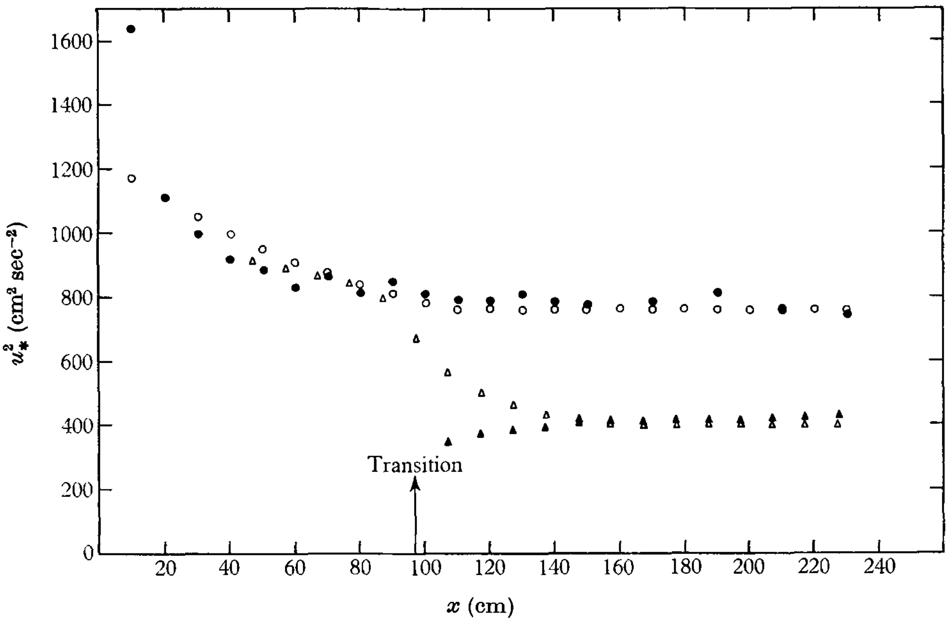

FIGURE 4. Measurements of u_*^2. By profile: case 1, ●; case 2, ▲. By momentum integral: case 1, ○; case 2, △.

Independent estimates of stress were also obtained in the form

$$\frac{\partial}{\partial x}\int_0^h u(u_0 - u)\,dz,$$

where u_0 and h are the velocity and height at the axis of the tunnel, it being assumed that u_* and $\partial u/\partial z$ are both zero at $z = h$ (Schlichting 1955). The term

$$\frac{\partial p}{\partial x}\int_0^h \{1 - (u/u_0)\}\,dz,$$

which represents the effect of the horizontal pressure gradient, was also measured and found to be negligible in comparison.

Figure 4 shows the stresses as assessed by the two methods. The satisfactory agreement between them unfortunately breaks down just where the information is most needed—close to the transition. This is only to be expected. The estimation by momentum integral depends on drawing a smooth curve through plotted values of

$$\int_0^h u(u_0 - u)\,dz$$

and must inevitably smooth out any transition. The usefulness of this method is simply in establishing the validity of the profile method by comparing them in regions where both may be expected to apply. The evidence thus suggests that some variation in surface stress does occur after the transition but that it is not nearly as great as indicated in the theory of Glaser *et al.* (1957).

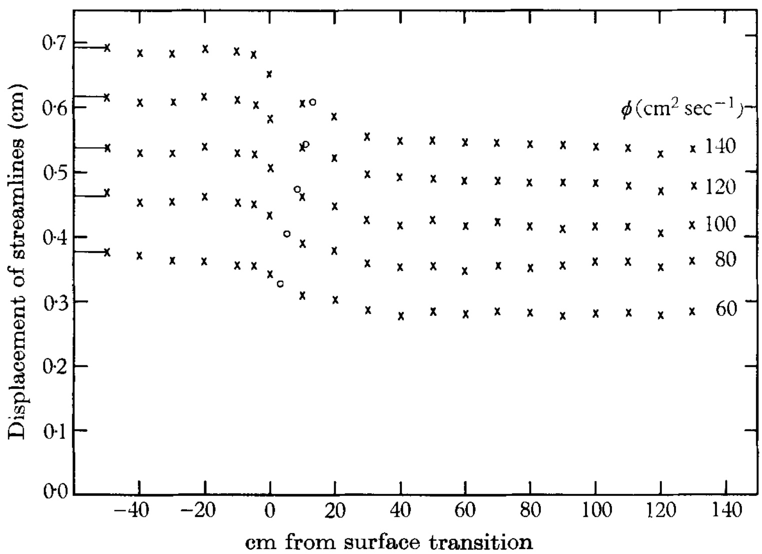

FIGURE 5. Displacement of streamlines from case 1 to case 2; ×, measured displacement; ○, values of $\eta(\frac{1}{2}z' + \frac{1}{2}z'')$. The displacements are plotted from arbitrary zeros (marked by short horizontal lines on the left) which represent the actual final heights attained by the corresponding streamlines in case 1.

Figure 5 shows (crosses) the displacement of the streamlines in case 2 from their case 1 positions as a function of x (measured from the transition, positive downwind). The short horizontal lines at the left show the actual final heights reached by the streamlines in case 1 and these have been used as arbitrary zeros in plotting streamline displacements. It is worth noting that the upper streamlines are the more reliable as here the necessary interpolation of velocity from the lowest measurement to zero at the surface carries least weight.

Values of η were calculated from equation (4) using profiles well upstream and downstream of the transition to determine

$$\int_0^{\phi} (u'' - u')\, d\phi$$

and taking the constant value of 366 cm^2 sec^{-2} for $u_*'^2 - u_*''^2$. The latter value was derived from the averages of the ultimate stresses as determined by momentum integral and profile methods. These values of η are plotted (circles) on figure 5 at heights midway between the displaced and undisplaced streamline heights. These points appear to indicate the centre of the transition rather well and provide further justification for making calculations on the basis of u_*' constant right up to the transition and then immediately superseded by a constant u_*''. Figure 5 also suggests that the streamlines begin to drop right at the surface transition, or even a little before it, so that the line AC of figure 1 should be approximately vertical. This feature is also apparent in the observations shown in figure 3.

The roughness length as determined by logarithmic profile for the rougher surface was 0·013 cm. We can also fit this profile to the flow over the smoother surface and obtain an equivalent z_0 of 0·00057 cm. If it is accepted that u_0 (which changed little from rough to smooth surface) is the analogue of u_g in the atmosphere, the observed values of $u'_* = 27{\cdot}8\,\mathrm{cm\,sec^{-1}}$ and $u''_* = 20{\cdot}2\,\mathrm{cm\,sec^{-1}}$ would give $n = 0{\cdot}102$.

5. Conclusion

We now have four estimates of n: 0·066 from the theories of Rossby & Montgomery and of Ellison, 0·058 from the observations of Lettau & Davidson, 0·138 from Deacon's observations and 0·102 from the wind tunnel. At present we can do no better than take the mean of these, 0·09, as the best available estimate of n. The value, 0·102, derived from the wind tunnel probably deserves less weight than the others since, in case 2, the flow over the downwind part of the surface was aerodynamically smooth and z''_0 did not really exist. However, even the complete omission of this value does not change the mean n to the accuracy given. Accepting $n = 0{\cdot}09$ and making use of equations (5), (6) and (7) which express the logarithmic profile, it is easy to show from (4) that

$$\frac{\eta}{z'_0} = \frac{1}{k^2(1-\alpha^{0\cdot 18})}\left\{\alpha^{1\cdot 18}\left(\frac{z''}{z''_0}\right)\left[\ln^2\left(\frac{z''}{z''_0}\right) - 2\ln\left(\frac{z''}{z''_0}\right) + 2\right] - \left(\frac{z'}{z'_0}\right)\left[\ln^2\left(\frac{z'}{z'_0}\right) - 2\ln\left(\frac{z'}{z'_0}\right) + 2\right]\right\}, \tag{8}$$

where $\alpha = z''_0/z'_0$. Since the velocity difference $u'' - u'$ must be taken at constant ϕ, we also have, from equation (6),

$$\alpha^{1\cdot 09}\left(\frac{z''}{z''_0}\right)\left\{\ln\left(\frac{z''}{z''_0}\right) - 1\right\} = \left(\frac{z'}{z'_0}\right)\left\{\ln\left(\frac{z'}{z'_0}\right) - 1\right\}. \tag{9}$$

Figure 6 shows (solid lines) η/z'_0, as determined from equations (8) and (9), plotted against z'/z'_0 for α equal to 10, 2, 0·5 and 0·1. It will be noted that, over the middle of the range, variation with α is not large and can, for practical purposes, be neglected. The broken line shows η/z'_0 calculated with n given the value 0·066 derived from the Rossby–Montgomery and Ellison theories ($\alpha = 0{\cdot}1$). As might be expected, variation with n is quite important and more work is required in order to establish its value beyond doubt.

It must be recalled that η represents the x-co-ordinate of a point somewhere in the middle of the transition zone. If the line AC of figure 1 is taken to be vertical and the displacement of the streamlines is assumed symmetrical about the points $x = \eta$, then the new velocity profile will be fully established to a given height only beyond a downwind distance of twice the η appropriate to that height. On the basis of figure 6, it might be suggested as a rough rule that a distance of 150 times the height of observation should be adequate in many practically useful situations.

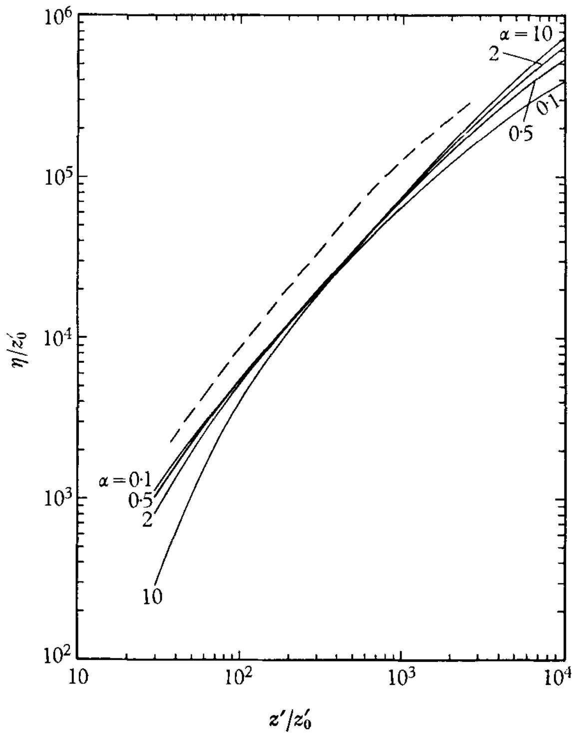

FIGURE 6. Values of η/z'_0.

I am indebted to Mr E. L. Deacon for permitting the use of the observations of figure 3. The wind-tunnel measurements, comprising over 700 separate velocity determinations, were carried out by Mr S. D. C. Wagg.

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